Optimal Joint Provision of Backhaul and Radio Access Networks

Zhi-Quan (Tom) Luo

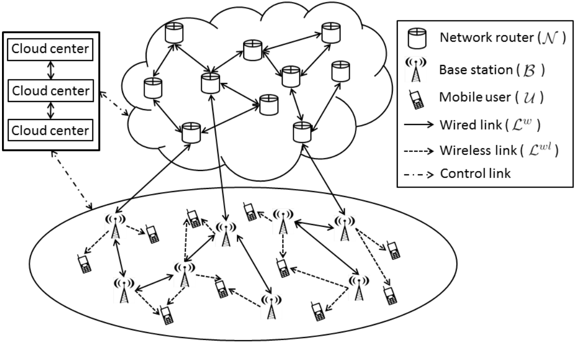
Joint work with Wei-cheng Liao, Mingyi Hong, Ruoyu Sun, Meisam Razaviyayn, Hang Zhang, Hadi Baligh

University of Minnesota

IEEE CTW, May 25-28, 2014, Curacao

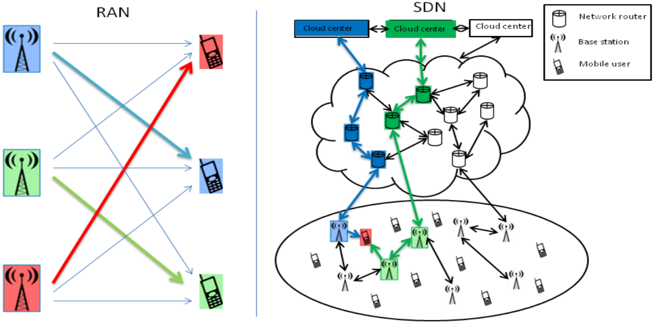
5G and Beyond: Key Features

- Cell-less deployment of radio access network (RAN)
- A large number of heterogeneous base stations connected via a backhaul network
- Virtualization: Software-defined, cloud-based provision of the backhaul and RAN



Main Issues: downlink case

- **RAN:** user-base station association, coordinated beamforming for interference mitigation
- **Backhaul:** multicommodity traffic engineering with capacitated links
- **Joint provision and why:** user-base station association
  - affects Backhaul: where to route the flow
  - affects RAN: direct link vs. interfering links



Joint Provision of RAN and Backhaul

In this talk, we describe an algorithmic approach (similar to those used for **BIGDATA**)

- Tailored for large-scale SDN with both wired and wireless links
- Achieves high system resource utilization
- Well suited for distributed/parallel implementation

Approach: integration of two existing algorithms

- The WMMSE algorithm for interference management in **RAN**
- The ADMM algorithm for traffic engineering in **Backhaul**

Road Map

- Resource management for RAN
- Traffic engineering for Backhaul
- Joint provision
- Simulations ⇒ joint provision can provide 3x gain

Interfering Broadcast Channel (IBC)

- K cell MIMO IBC (multicell downlink)
- Each base station k serves I_k number of users in cell k
- The Tx k uses the beamformer $\mathbf{V}_{i_k}$ to send the signal to Rx i in cell k

$$\mathbf{x}_k = \sum_{i=1}^{I_k} \mathbf{V}_{i_k} \mathbf{s}_{i_k}$$

- The received signal of the i_k-th user in cell k:

$$\mathbf{y}_{i_k} = \mathbf{H}_{i_k k} \mathbf{V}_{i_k} \mathbf{s}_{i_k} + \sum_{\ell \neq i} \mathbf{H}_{\ell_k k} \mathbf{V}_{\ell_k} \mathbf{s}_{\ell_k} + \sum_{j \neq k} \sum_{\ell=1}^{I_j} \mathbf{H}_{i_k j} \mathbf{V}_{\ell_j} \mathbf{s}_{\ell_j} + \mathbf{n}_{i_k}$$

- $\mathbf{H}_{i_k j}$: the channel matrix from Tx j to the Rx i in cell k
- Interacell and intercell interference

General utility maximization

- Sum-utility maximization problem:

$$
\begin{aligned}
\max_{\{\mathbf{V}\}} \quad & \sum_{k=1}^{K} \sum_{i_k=1}^{I_k} u_{i_k}(R_{i_k}) \\
\text{s.t.} \quad & \sum_{i=1}^{I_k} \mathrm{Tr}(\mathbf{V}_{i_k}\mathbf{V}_{i_k}^H) \leq P_k, \;\; \forall\, k = 1, 2, \ldots, K
\end{aligned}
\qquad \text{(P)}
$$

- The rate function (define $\mathbf{Q}_{i_k} \triangleq \mathbf{V}_{i_k}\mathbf{V}_{i_k}^H$):

$$
R_{i_k} \triangleq \log\det\left(\mathbf{I}+\mathbf{H}_{i_k k}\mathbf{Q}_{i_k}\mathbf{H}_{i_k k}^H\left(\sum_{(\ell,j)\neq(i,k)} \mathbf{H}_{i_k j}\mathbf{Q}_{\ell_j}\mathbf{H}_{i_k j}^H+\sigma_{i_k}^2\mathbf{I}\right)^{-1}\right)
$$

System Utilities

- Consider α-fairness utility functions
- For $\alpha \geq 0$, it is defined as follows

$$U_\alpha(R_1, \cdots, R_K) = \begin{cases} \sum_{k=1}^{K} \frac{R_k^{1-\alpha}}{1-\alpha} & \text{if } \alpha \neq 1; \\ \sum_{k=1}^{K} \log(R_k) & \text{if } \alpha = 1. \end{cases} \quad (1)$$

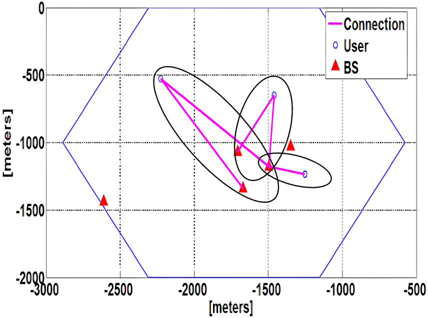
- **Special cases:** sum-rate, proportionally fair rate, harmonic mean rate, max-min rate etc

Joint BS Association and Transceiver Design

- Two Goals:
  - for each user i_k, identify a small set of serving BSs $\mathcal{S}_{i_k} \subseteq \mathcal{Q}_k$;
  - optimize transmit beamformers $\{\mathbf{v}_{i_k}^{q_k}\}_{q_k \in \mathcal{S}_{i_k}, i_k \in \mathcal{I}_k}$

- $|\mathcal{S}_{i_k}|$ is small $\Longrightarrow$ $\mathbf{v}_{i_k} \triangleq \left[(\mathbf{v}_{i_k}^1)^H, \cdots, (\mathbf{v}_{i_k}^{Q_k})^H\right]^H$ should contain only a few nonzero blocks

- Sparse utility maximization!



Utility Maximization for Joint Clustering/Precoder Design

- The beamforming vector $\mathbf{v}_{i_k}$ should be group sparse $\Rightarrow$ nonsmooth regularization.
- A utility maximization problem **[Hong et.al. 2013]**

$$\begin{aligned} \max_{\{\mathbf{v}_{i_k}^{q_k}\}} \quad & \sum_{k\in\mathcal{K}} \left(\sum_{i_k\in\mathcal{I}_k} u_{i_k}(R_{i_k}) \right) + \lambda \sum_{k\in\mathcal{K}, q_k\in\mathcal{Q}_k} \|\mathbf{v}_{i_k}^{q_k}\|_2 \\ \text{s.t.} \quad & \sum_{i_k\in\mathcal{I}_k} (\mathbf{v}_{i_k}^{q_k})^H \mathbf{v}_{i_k}^{q_k} \le P_{q_k}, \ \forall\, q_k \in \mathcal{Q}_k, \ \forall\, k \in \mathcal{K} \end{aligned} \qquad (P_1)$$

- User i_k served by one BS $\Leftrightarrow$ $\mathbf{v}_{i_k}$ has one nonzero block $\Leftrightarrow$ $|\mathcal{S}_{i_k}| = 1$.

Interference Management via Utility Maximization

- An area of active research, many algorithms have been proposed:
  - Game theory based, best response
  - Successive convex approximation
  - Pricing based, uplink-downlink duality
  - Distributed/parallel, Gauss-Seidel or Jacobi
  - Geometric programming, sparse optimization, stochastic incremental

- Many many contributors:

S. Barbarossa	M. Bengtsson	R. Berry	E. Bjornson	H. Boche	M. Chiang
R. Cui	D. Gesbert	G. Giannakis	A. Goldsmith	R. Heath	M. Honig
E. Jorswieck	M. Juntti	E. Larsson	V. Lau	K. Ma	M. Moonen
B. Ottersten	D. Palomar	J.S. Pang	A. Paulraj	M. Pesavento	A. Petropulu
V. Poor	T. Quek	M. Schubert	G. Scutari	N. Sidiropoulos	S. Stanczak
A. Tolli	W. Utschick	L. Vandendorp	S. Vorobyov	W. Yu	R. Zhang

......

Complexity Analysis

1. *Sum utility maximization for IBC* **[L.-Zhang'08]**
   - $K = 1$, M, N arbitrary: convex opt. (e.g., water-filling)
   - K arbitrary, $\min\{M, N\} \geq 3$: NP-hard
2. *Joint user-BS association and precoder design* **[Hong et.al.'13]**

   Suppose $|\mathcal{S}_{i_k}| = 1$ for all i_k and utility function is $U_\alpha(\cdot)$. The system level problem (P_1) is NP-hard when

   - either $\alpha = 0$ (the Sum-Rate utility function);
   - or $\min(M, N) \geq 3$

A Polynomial Time Solvable Case

- Consider the following network setting
  - $K = B$, i.e., the number of BSs and the number of users are the same
  - $M_b = N_k = 1 \;\forall\; b,\; k$, i.e., both the BSs and users have a single antenna
  - Each BS can only serve a single user
- The objective: maximize the minimum transmission rate (the min-rate utility function)

A Special Case (Cont.)

- In the above setting, the problem becomes a joint user-BS matching and power allocation problem
- Let $\mathbf{p} = [p_1 \cdots, p_B]$ denote the BSs' transmission power
- the optimization problem is

$$\begin{aligned} \max_{\mathbf{p},\mathbf{a}} \quad & \min_{k=1,\ldots,K} \{R_k\}, \\ \text{s.t.} \quad & 0 \le p_b \le P_b, b = 1, \cdots, B \\ & \frac{|H_{k\mathbf{a}_k}|^2 p_{\mathbf{a}_k}}{\sigma_k^2 + \sum_{b \neq \mathbf{a}_k} |H_{kb}|^2 p_b} \ge 1, k = 1, \ldots, K \\ & \mathbf{a}_k \neq \mathbf{a}_l, \ \forall\ k \neq l. \end{aligned} \tag{2}$$

- Related work: Rashid-Farrokhi et.al.'97, '98; Hanly'95

A Special Case (Cont.)

- **Result:** if (2) is feasible, then
  - the optimal association can be found via maximum weighted matching
  - the weights are $\{\log |H_{kb}|^2\}_{(k,b)\in\mathcal{K}\times\mathcal{B}}$
- *Algorithm:*
  Step 1: solve the maximum weighted matching problem to obtain $\mathbf{a}^*$;
  Step 2: fix $\mathbf{a} = \mathbf{a}^*$, solve a max-min SIR balancing problem to find optimal $\mathbf{p}^*$

Max-min Fair Joint BS Assignment and Power Control

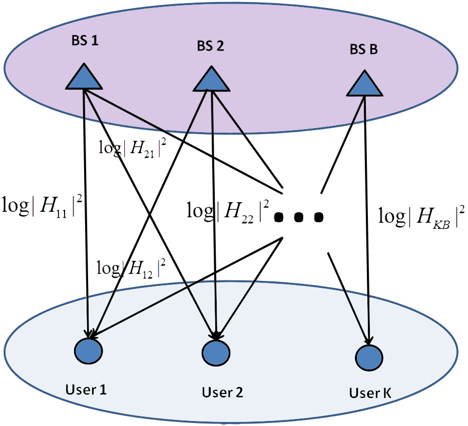


Figure: BS association via Max-$\log$(weight) Matching

Two Commonly Used Utilities

- Weighted sum–rate maximization:

$$\begin{aligned} \max_{\{\mathbf{V}\}} \quad & \sum_{k=1}^{K} \sum_{i_k=1}^{I_k} \alpha_{i_k} R_{i_k} \\ \text{s.t.} \quad & \sum_{i=1}^{I_k} \mathrm{Tr}(\mathbf{V}_{i_k} \mathbf{V}_{i_k}^H) \le P_k, \;\; \forall\, k = 1, 2, \ldots, K \end{aligned} \tag{3}$$

- Sum–MSE minimization:

$$\begin{aligned} \min_{\{\mathbf{U},\mathbf{V}\}} \quad & \sum_{k=1}^{K} \sum_{i=1}^{I_k} \alpha_{i_k} \mathrm{Tr}(\mathbf{E}_{i_k}) \\ \text{s.t.} \quad & \sum_{i=1}^{I_k} \mathrm{Tr}(\mathbf{V}_{i_k} \mathbf{V}_{i_k}^H) \le P_k, \;\; \forall\, k = 1, 2, \ldots, K \end{aligned}$$

Two Commonly Used Utilities

- Weighted sum–rate maximization:

$$
\begin{aligned}
\max_{\{\mathbf{V}\}} \quad & \sum_{k=1}^{K} \sum_{i_k=1}^{I_k} \alpha_{i_k} R_{i_k} \\
\text{s.t.} \quad & \sum_{i=1}^{I_k} \mathrm{Tr}(\mathbf{V}_{i_k} \mathbf{V}_{i_k}^H) \leq P_k, \ \ \forall\, k = 1, 2, \ldots, K
\end{aligned}
\tag{4}
$$

- Weighted sum–MSE minimization:

$$
\begin{aligned}
\min_{\{\mathbf{U},\mathbf{V}\}} \quad & \sum_{k=1}^{K} \sum_{i=1}^{I_k} \alpha_{i_k} \mathrm{Tr}(\mathbf{W}^* \mathbf{E}_{i_k}) \\
\text{s.t.} \quad & \sum_{i=1}^{I_k} \mathrm{Tr}(\mathbf{V}_{i_k} \mathbf{V}_{i_k}^H) \leq P_k, \ \ \forall\, k = 1, 2, \ldots, K
\end{aligned}
$$

Two commonly used utilities (cont.)

$$\begin{aligned}\mathbf{E}_{i_k} &\triangleq \mathbb{E}_{\mathbf{s},\mathbf{n}}\left[(\hat{\mathbf{s}}_{i_k} - \mathbf{s}_{i_k})(\hat{\mathbf{s}}_{i_k} - \mathbf{s}_{i_k})^H\right] \\ &= (\mathbf{I} - \mathbf{U}_{i_k}^H \mathbf{H}_{i_k k} \mathbf{V}_{i_k})(\mathbf{I} - \mathbf{U}_{i_k}^H \mathbf{H}_{i_k k} \mathbf{V}_{i_k})^H \\ &+ \sum_{(\ell,j)\neq(i,k)} \mathbf{U}_{i_k} \mathbf{H}_{i_k j} \mathbf{V}_{\ell_j} \mathbf{V}_{\ell_j}^H \mathbf{H}_{i_k j}^H \mathbf{U}_{i_k}^H + \sigma_{i_k}^2 \mathbf{U}_{i_k}^H \mathbf{U}_{i_k},\end{aligned}$$

- The well known MMSE receiver:

$$\mathbf{U}_{i_k}^{\mathrm{mmse}} = \mathbf{J}_{i_k}^{-1} \mathbf{H}_{i_k k} \mathbf{V}_{i_k},$$

where $\mathbf{J}_{i_k} \triangleq \sum_{j=1}^{K} \sum_{\ell=1}^{I_j} \mathbf{H}_{i_k j} \mathbf{V}_{\ell_j} \mathbf{V}_{\ell_j}^H \mathbf{H}_{i_k j}^H + \sigma_{i_k}^2 \mathbf{I}$.

- Using the MMSE receiver leads to the MMSE matrix:

$$\mathbf{E}_{i_k}^{\mathrm{mmse}} = \mathbf{I} - \mathbf{V}_{i_k}^H \mathbf{H}_{i_k k}^H \mathbf{J}_{i_k}^{-1} \mathbf{H}_{i_k k} \mathbf{V}_{i_k}.$$

- We have $R_{i_k} = \log\det\left(\left(\mathbf{E}_{i_k}^{\mathrm{mmse}}\right)^{-1}\right)$

A matrix weighted MMSE problem

Theorem: *Let* $\mathbf{W}_{i_k} \succeq \mathbf{0}$ *be the weight matrix for receiver* i_k*. Then, the optimization problem*

$$\begin{aligned} \min_{\{\mathbf{W},\mathbf{U},\mathbf{V}\}} \quad & \sum_{k=1}^{K}\sum_{i=1}^{I_k} \alpha_{i_k}\left(\mathrm{Tr}(\mathbf{W}_{i_k}\mathbf{E}_{i_k}) - \log\det(\mathbf{W}_{i_k})\right) \\ \text{s.t.} \quad & \sum_{i=1}^{I_k} \mathrm{Tr}(\mathbf{V}_{i_k}\mathbf{V}_{i_k}^H) \le P_k,\ \forall\ k = 1, 2, \ldots, K \end{aligned} \tag{5}$$

is equivalent to the weighted sum-rate maximization problem (3)*.*

- Equivalence means they have the same local/global minimizers.
- An extension of the WMMSE algorithm for the BC channel (S. Christensen, R. Agarwal, etc., IEEE TWC'08)

The WMMSE algorithm

- **WMMSE algorithm:** solve (5) by the block coordinate descent algorithm
- Closed form updates at each iteration
- Subproblems are decomposed completely across users
- We prove any limit point of the WMMSE algorithm is a stationary point of the weighted sum rate maximization problem (3)

The pseudocode of the WMMSE algorithm

Initialize $\mathbf{V}_{i_k}$'s such that $\mathrm{Tr}\left(\mathbf{V}_{i_k}\mathbf{V}_{i_k}^H\right) = \frac{p_k}{I_k}$

repeat

$\quad \mathbf{W}'_{i_k} \leftarrow \mathbf{W}_{i_k}, \quad \forall\, i_k \in \mathcal{I}$

$\quad \mathbf{U}_{i_k} \leftarrow \left(\sum_{(j,\ell)} \mathbf{H}_{i_k j}\mathbf{V}_{\ell_j}\mathbf{V}_{\ell_j}^H\mathbf{H}_{i_k j}^H + \sigma_{i_k}^2\mathbf{I}\right)^{-1}\mathbf{H}_{i_k k}\mathbf{V}_k,\ \forall\, i_k \in \mathcal{I}$

$\quad \mathbf{W}_{i_k} \leftarrow \left(\mathbf{I} - \mathbf{U}_{i_k}^H\mathbf{H}_{i_k k}\mathbf{V}_{i_k}\right)^{-1},\ \forall\, i_k \in \mathcal{I}$

$\quad \mathbf{V}_{i_k} \leftarrow$

$$\alpha_{i_k}\left(\sum_{(j,\ell)}\alpha_{\ell_j}\mathbf{H}_{\ell_j k}^H\mathbf{U}_{\ell_j}\mathbf{W}_{\ell_j}\mathbf{U}_{\ell_j}^H\mathbf{H}_{\ell_j k} + \mu_k^*\mathbf{I}\right)^{-1}\mathbf{H}_{i_k k}^H\mathbf{U}_{i_k}\mathbf{W}_{i_k},\ \forall\, i_k$$

until $\left|\sum_{(j,\ell)}\log\det\left(\mathbf{W}_{\ell_j}\right) - \sum_{(j,\ell)}\log\det\left(\mathbf{W}'_{\ell_j}\right)\right| \leq \epsilon$

Note: no parameters to tune!

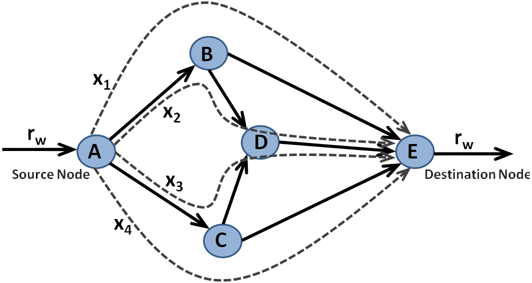
Extensions of the WMMSE Approach

WMMSE algorithm is quite flexible. Several extensions are possible.

- deal with general utility functions
- joint user grouping and transceiver design
- joint base station association/activation and transceiver design
- partial CSI: **stochastic WMMSE** for expected sum-rate maximization

TE for the Backhaul

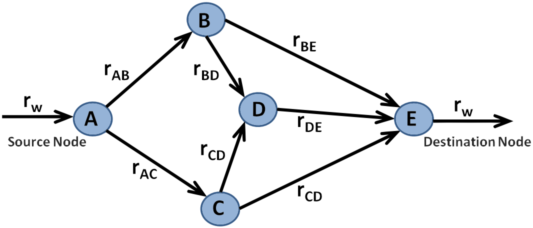
- Two formulations for traffic engineering (TE) without interference [Bertsekas 87,88]
- i) **Path-flow formulation:** Paths are predetermined (dash lines)



  - Flow rate on each path is nonnegative, i.e., $x_1 \sim x_4 \geq 0$
  - Flow rate for this source-destination pair $r_w = \sum_{i=1}^{4} x_i$
  - (Pros) Suitable for small/medium network
  - (Cons) Possible number of paths grow exponentially

Existing Works (cont.)

- ii) **Link-flow formulation:** Paths are implicitly determined



  - Flow rate on each link is nonnegative, i.e., $\forall\ r_{ij} \geq 0$, $i, j \in \{A \sim E\}$
  - Each node satisfies flow rate balance equation, e.g., for node D $r_{BD} + r_{CD} = r_{DE}$
  - (Pros) Suitable for large network (# of variables grows linearly)
  - (Cons) Result in undesirably large number of flow paths

Existing Works (cont.)

- TE in the presence of wireless interference is much more challenging because
  - Link capacity is not fixed
  - Existence of multiuser interference
  - Existence of multiple parallel channels (or multiple antennas)
- Cross-layer network utility maximization problem considers the joint routing and resource optimization [Shroff 06][Chiang 07][Xiao 04]
  - (Approximate) no interference
  - Dual decomposition $\Rightarrow$ slow convergence

The Problem

- Consider a backhaul with no interference (e.g., only wired links or highly directional wireless links)
- The nodes of this backhaul, $\mathcal{V}$, consist of the set of network routers $\mathcal{N}$, the set of BSs $\mathcal{B}$
- The set of directed links:

$$\mathcal{L} \triangleq \left\{(s,d) \mid (s,d) \in \mathcal{L},\ \forall s,d \in \mathcal{N} \bigcup \mathcal{B}\right\}$$

- Let $r_l(m)$ denote the flow on link l for commodity m.
- Each link $l \in \mathcal{L}$ has a fixed capacity C_l:

$$\sum_{m=1}^{M} r_l(m) \leq C_l \tag{6}$$

The Problem (cont.)

- **Task**: route M commodities using M flows, each with rate r_m, $m = 1, \ldots, M$.
- Each commodity m is associated with a source-destination pair $(S(m), D(m))$
- Flow conservation constraint: per node/flow

$$\underbrace{\sum_{l \in \mathrm{In}(v)} r_l(m) + 1_{\{S(m)\}}(v) r_m}_{\text{incomming flow}} = \underbrace{\sum_{l \in \mathrm{Out}(v)} r_l(m) + 1_{\{D(m)\}}(v) r_m}_{\text{outgoing flow}}, \quad \forall\ m, v \in \mathcal{V} \qquad (7)$$

where $\mathrm{In}(b)$ (resp. $\mathrm{Out}(b)$) are the set of links that go into (resp. come out of) node b.

The Problem (cont.)

- Use the minimum rate as our optimization criterion:

$$\max \min_{m} r_m, \quad \text{s.t. (6) and (7)}$$

which is equivalent to a large linear program (LP)

$$\boxed{\begin{array}{lll} (\mathcal{P}): & \max & r \\ & \text{s.t.} & r \geq 0,\ r_m \geq r,\ r_l(m) \geq 0,\ \forall\, m,\ \forall\, l \in \mathcal{L}, \\ & & \text{(6) and (7)} \end{array}}$$

- Variables: $M|\mathcal{L}|$, constraints: $|\mathcal{L}| + M|\mathcal{V}|$; general purpose LP solvers can be quite slow
- **Customized Algorithm**: decompose $(\mathcal{P})$ into simple subproblems of small sizes, and solve in parallel $\Rightarrow$ **ADMM**!

Brief Review of ADMM Algorithm

- The ADMM was developed in 1970s; recently popular for large-scale optimization [Boyd 11]
- The ADMM solves the following structured convex problem

$$\min\ f(\mathbf{x}) + g(\mathbf{z}) \tag{8}$$
$$\text{s.t. } \mathbf{Ax} + \mathbf{Bz} = \mathbf{c},\ \mathbf{x} \in \mathcal{C}_1,\ \mathbf{z} \in \mathcal{C}_2 \tag{9}$$

- First introduce a quadratic penalization term $(\rho/2)\|\mathbf{Ax} + \mathbf{Bz} - \mathbf{z}\|^2$, $\rho > 0$, to the objective function:

$$\min\ f(\mathbf{x}) + g(\mathbf{z}) + (\rho/2)\|\mathbf{Ax} + \mathbf{Bz} - \mathbf{c}\|^2$$
$$\text{s.t. } \mathbf{Ax} + \mathbf{Bz} = \mathbf{c},\ \mathbf{x} \in \mathcal{C}_1,\ \mathbf{z} \in \mathcal{C}_2.$$

The ADMM Algorithm

- The Lagrangian function for the penalized problem is

$$L_\rho(\mathbf{x},\mathbf{z},\mathbf{y}) = f(\mathbf{x}) + g(\mathbf{z}) + \mathbf{y}^T(\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{c}) \\ + (\rho/2)\|\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{c}\|^2$$

- The primal problem is given by

$$d(y) = \min_{\mathbf{x}\in\mathcal{C}_1,\mathbf{z}\in\mathcal{C}_2} L_\rho(\mathbf{x},\mathbf{z},\mathbf{y}) \tag{10}$$

- The dual problem is

$$d^* = \max_{y} d(\mathbf{y}), \tag{11}$$

d^* equals the optimal value of (8) under mild conditions

The Dual Ascent Algorithm

Dual Ascent Algorithm

At each iteration $t \geq 1$, first update the primal variable x and then update the dual multiplier:

$$\begin{cases} (\mathbf{x}^{t+1}, \mathbf{z}^{t+1}) = \arg\min_{\mathbf{x}\in\mathcal{C}_1, \mathbf{z}\in\mathcal{C}_2} L(\mathbf{x}, \mathbf{z}; \mathbf{y}^t) \\ \quad = \arg\min_{\mathbf{x}\in\mathcal{C}_1, \mathbf{z}\in\mathcal{C}_2} f(\mathbf{x}) + g(\mathbf{z}) + \langle y^t, \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{c}\rangle + \frac{\rho}{2}\|\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{c}\|^2, \\ \mathbf{y}^{t+1} = \mathbf{y}^t + \alpha(\mathbf{A}\mathbf{x}^{t+1} + \mathbf{B}\mathbf{z}^{t+1} - \mathbf{c}), \end{cases}$$

where $\alpha > 0$ is the step size for the dual update.

- Note: $\nabla d(\mathbf{y}^t) = \mathbf{A}\mathbf{x}^{t+1} + \mathbf{B}\mathbf{z}^{t+1} - \mathbf{c} \Rightarrow$ dual ascent $d(\mathbf{y}^{t+1}) \geq d(\mathbf{y}^t)$.
- However, the minimization (primal step) can be difficult.
- Since the objective is separable, we may perform the primal step *inexactly using block coordinate descent*..., $\Rightarrow$ **ADMM!**

The ADMM Algorithm

Alternating Direction Method of Multipliers (ADMM)

At each iteration $r \geq 1$, first update the primal variable blocks in the Gauss-Seidel fashion and then update the dual multiplier:

$$\begin{cases} \mathbf{x}^{t+1} = \arg\min_{\mathbf{x} \in \mathcal{C}_1} L(\mathbf{x}, \mathbf{z}^r; \mathbf{y}^t), \\ \mathbf{z}^{t+1} = \arg\min_{\mathbf{z} \in \mathcal{C}_2} L(\mathbf{x}^{t+1}, \mathbf{z}; \mathbf{y}^t), \\ \mathbf{y}^{t+1} = \mathbf{y}^t + \alpha(\mathbf{A}\mathbf{x}^{t+1} + \mathbf{B}\mathbf{z}^{t+1} - \mathbf{c}), \end{cases}$$

where $\alpha > 0$ is the step size for the dual update.

- Inexact primal minimization $\Rightarrow$ $(\mathbf{A}\mathbf{x}^{t+1} + \mathbf{B}\mathbf{z}^{t+1} - \mathbf{c})$ is no longer the dual gradient!
- Dual ascent property $d(\mathbf{y}^{t+1}) \geq d(\mathbf{y}^t)$ is lost $\Rightarrow$ complications in the convergence analysis
- Consider $\alpha = 0$ or ≈ 0 ...

The ADMM Algorithm (cont.)

- The Alternating Direction Method of Multipliers (ADMM) optimizes the augmented Lagrangian function one block variable at each time [Hong-Luo 12, Bertsekas 10]
- Recently found lots of applications in large-scale structured optimization; see [Boyd 11] for a survey
- Highly efficient, especially when the per-block subproblems are easy to solve (with closed-form solution)
- If the optimal solution set is non-empty, and if $\mathbf{A}^T\mathbf{A}$ and $\mathbf{B}^T\mathbf{B}$ are invertible, then every limit point of $\{\mathbf{x}^k, \mathbf{z}^k\}$ is an optimal solution
- The convergence rate of ADMM can be enhanced via dynamically adjusting ρ or over-relaxation

An ADMM Approach for Multi-commodity Routing

- Apply ADMM to solve coupled problem $(\mathcal{P})$ with easy and parallel subproblems
- To decouple the elements of $\mathcal{V}$ from the conservation constraints (7), the following slack variables are introduced

$$r = \hat{r} \tag{12a}$$

$$r_m = \hat{r}_m^{S(m)},\ r_m = \hat{r}_m^{D(m)},\quad \forall\, m = 1 \sim M, \tag{12b}$$

$$r_l(m) = \hat{r}_l^s(m),\ r_l(m) = \hat{r}_l^d(m),\\ \forall\, l = (s,d) \in \mathcal{L},\ m = 1 \sim M, \tag{12c}$$

- Group the optimization variables into two variable sets

$$\mathbf{r} \triangleq \{r, r_m, r_l(m) \mid m = 1 \sim M,\ \forall\, l \in \mathcal{L}\}$$

$$\hat{\mathbf{r}} \triangleq \left\{\hat{r}, \hat{r}_m^{S(m)}, \hat{r}_m^{D(m)}, \hat{r}_l^s(m), \hat{r}_l^d(m) \mid m = 1 \sim M,\ \forall\, l = (s,d) \in \mathcal{L}\right\}$$

$\Rightarrow$ The constraints are then decoupled

An ADMM Approach for Multi-commodity Routing

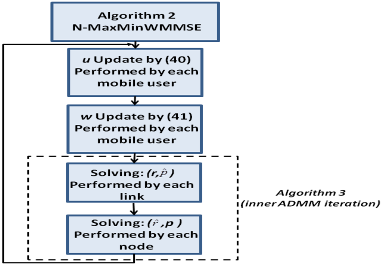
- Specifically, problem $(\mathcal{P})$ is equivalent to

$$
\begin{aligned}
(\hat{\mathcal{P}}): \ \max \ & (r+\hat{r})/2 \\
\text{s.t. } & r \geq 0, \ r_m \geq r, \ r_l(m) \geq 0, \ m = 1 \sim M, \\
& \sum_{m=1}^{M} r_l(m) \leq C_l, \ l \in \mathcal{L} \\
& \sum_{l \in \mathrm{In}(v)} \hat{r}_l^v(m) + 1_{\{S(m)\}}(v)\hat{r}_m^v \\
& = \sum_{l \in \mathrm{Out}(v)} \hat{r}_l^v(m) + 1_{\{D(m)\}}(v)\hat{r}_m^v, \ m = 1 \sim M, \ \forall v \in \mathcal{V}, \\
& \text{and (12).}
\end{aligned}
$$

- Objective function and constraints (except (12)) are separable over $\mathbf{r}$ and $\hat{\mathbf{r}}$
  $\Rightarrow$ Satisfy the structure of ADMM algorithm!

An ADMM Approach for Multi-commodity Routing

- ADMM approach Algorithm 1 is outlined as follows



where $\boldsymbol{\delta} \triangleq \{\delta, \delta_m^{S(m)}, \delta_m^{D(m)}, \delta_l^s(m), \delta_l^d(m) \mid m = 1 \sim M, \forall\, l = (s,d) \in \mathcal{L}\}$ is the dual variables for (12).

- **Theorem** Every limit point of the sequence $\{\mathbf{r}^{(k)}\}$ generated by Algorithm 1 is an optimal solution of problem $(\mathcal{P})$.
- Each step of Algorithm 1 will be discussed in details (iteration index will be dropped for simplicity)

Solving $\mathbf{r}$

- **The first step** (solving $\mathbf{r}$) can be decoupled into two parts
  - $\{r, r_m \mid m = 1 \sim M\}$
  - $\{r_l(m) \mid m = 1 \sim M, \forall\, l \in \mathcal{L}\}$
- Subproblem for $\{r, r_m \mid m = 1 \sim M\}$:

$$\max \frac{r}{2} - \frac{\rho_1}{2}\left(\hat{r} - r - \frac{\delta}{\rho_1}\right)^2 - \frac{\rho_1}{2}\sum_{m=1}^{M}\left[\left(\hat{r}_m^{S(m)} - r_m - \frac{\delta_m^{S(m)}}{\rho_1}\right)^2 + \left(\hat{r}_m^{D(m)} - r_m - \frac{\delta_m^{D(m)}}{\rho_1}\right)^2\right]$$

$$\text{s.t. } r_m \geq r,\ m = 1 \sim M,\ r \geq 0.$$

$\Rightarrow$ Solved by bisection search over $r \geq 0$.

Solving $\mathbf{r}$ (cont.)

- Subproblem for $\{r_l(m) \mid m = 1 \sim M, \forall\, l \in \mathcal{L}\}$:

  $\Rightarrow$ Decoupled over each link

  $\Rightarrow$ For link $l = (s, d) \in \mathcal{L}$, the following problem is solved

$$\min \sum_{m=1}^{M} \left[\left(\hat{r}_l^s(m) - r_l(m) - \frac{\delta_l^s(m)}{\rho_1} \right)^2 + \left(\hat{r}_l^d(m) - r_l(m) - \frac{\delta_l^d(m)}{\rho_1} \right)^2 \right]$$

$$\text{s.t.} \sum_{m=1}^{M} r_l(m) \le C_l,\ \ r_l(m) \ge 0,\ \ m = 1 \sim M.$$

  $\Rightarrow$ Efficiently solved via by bisection procedure.

Solving $\hat{\mathbf{r}}$ and Updating Dual Variables

- **The second step** (solving $\hat{\mathbf{r}}$) can be decoupled into two parts
  - $\{\hat{r}_m^{S(m)}, \hat{r}_m^{D(m)}, \hat{r}_l^s(m), \hat{r}_l^d(m)\}$ and $\hat{r}$
- Subproblem for $\{\hat{r}_m^{S(m)}, \hat{r}_m^{D(m)}, \hat{r}_l^s(m), \hat{r}_l^d(m)\}$:

$\Rightarrow$ Decouple across nodes. For node $v \in \mathcal{V}$, the subproblem is

$$\min \sum_{l \in \mathrm{In}(v) \bigcup \mathrm{Out}(v)} \left(\hat{r}_l^v(m) - r_l(m) - \frac{\delta_l^v(m)}{\rho_1}\right)^2 + 1_{\{S(m),D(m)\}}(v)\left(\hat{r}_m^v - r_m - \frac{\delta_m^v}{\rho_1}\right)^2$$

$$\text{s.t.} \sum_{l \in \mathrm{In}(v)} \hat{r}_l^v(m) + 1_{\{S(m)\}}(v)\hat{r}_m^v = \sum_{l \in \mathrm{Out}(v)} \hat{r}_l^v(m) + 1_{\{D(m)\}}(v)\hat{r}_m^v$$

$\Rightarrow$ One equality constraint only $\Rightarrow$ Closed-form solution

Solving $\hat{\mathbf{r}}$ and Updating Dual Variables (cont.)

- Subproblem for $\hat{r}$: An easy unconstraint quadratic problem

$$\max\ \hat{r}/2 - (\rho_1/2)\left(\hat{r} - r - \frac{\delta}{\rho_1}\right)^2 \Rightarrow\ \hat{r}^\star = r + \frac{1+2\delta}{2\rho_1}.$$

- **The third step** (update the Lagrange multipliers $\boldsymbol{\delta}^{(t+1)}$) is given by

$$\begin{aligned}
\delta^{(k+1)} &= \delta^{(k)} - \rho_1(\hat{r}^{(k+1)} - r^{(k+1)}),\\
\delta_m^{S(m)(k+1)} &= \delta_m^{S(m)(k)} - \rho_1(\hat{r}_m^{S(m)(k+1)} - r_m^{(k+1)}),\\
\delta_m^{D(m)(k+1)} &= \delta_m^{D(m)(k)} - \rho_1(\hat{r}_m^{D(m)(k+1)} - r_m^{(k+1)}),\\
\delta_l^{s(k+1)}(m) &= \delta_l^{s(k)}(m) - \rho_1(\hat{r}_l^{s(k+1)}(m) - r_l^{(k+1)}(m)),\\
\delta_l^{d(k+1)}(m) &= \delta_l^{d(k)}(m) - \rho_1(\hat{r}_l^{d(k+1)}(m) - r_l^{(k+1)}(m)),\\
&\qquad m = 1 \sim M,\ \forall l = (s,d) \in \mathcal{L}
\end{aligned}$$

- They can be updated locally by each node

Implementation Issues for Algorithm 1

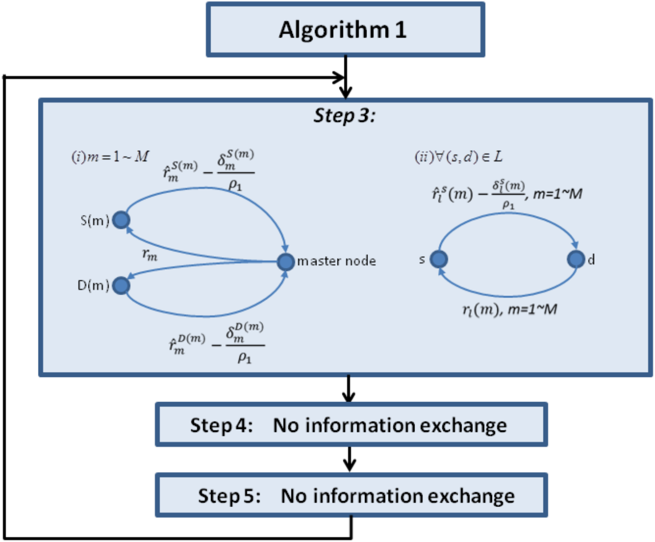
- Low complexity, scales well with the network size

  Each step is (semi)closed-form and solvable in parallel across links/nodes

  The per-iteration complexity is in the order of $\mathcal{O}(|\mathcal{L}| + |\mathcal{V}|)$.

- Distributed implementation
  - All computation can be distributed to the nodes
  - A single master node that can communicate with all source and destination nodes is needed
  - The neighboring nodes exchange $2M$ real symbols in each iteration

Implementation of Algorithm 1 (cont.)



Implementation of Algorithm 1 with Zones of Nodes

- For SDN, each cloud center manages a subset of nodes within geographical zone
  - centralized computation within each zone
  - distributed computation/message exchange between zones
- Denote $v \in \mathcal{Z}_i$ if node v is within the ith zone
- Modify variable splitting procedure:

$$r_l(m) = \hat{r}_l^s(m),\ r_l(m) = \hat{r}_l^d(m), \quad \forall\, l = (s \in \mathcal{Z}_i, d \in \mathcal{Z}_j) \in \mathcal{L}, \quad i \neq j,\ m = 1 \sim M,$$

$$r_l(m) = \hat{r}_l(m), \quad \forall\, l = (s \in \mathcal{Z}_i, d \in \mathcal{Z}_j) \in \mathcal{L},\ i = j,\ m = 1 \sim M,$$

i.e., only the link rates on the boundary are split

- Links belong to bordering links (BD) or interior links (IT)

$$\mathrm{BD}_i \triangleq \{l = (s,v),\ l = (v,d) \in \mathcal{L} \mid \forall\, v \in \mathcal{Z}_i,\ s,d \in \mathcal{Z}_k,\ k \neq i\}$$

$$\mathrm{IT}_i \triangleq \{l = (s,v),\ l = (v,d) \in \mathcal{L} \mid \forall s,d,v \in \mathcal{Z}_i\}$$

Implementation of Algorithm 1 with Zones of Nodes

- Similar ADMM subproblems except the second step for zone i:

$$
\begin{aligned}
\min \sum_{l \in \mathrm{BD}_i} & \left(\hat{r}_l^v(m) - r_l(m) - \frac{\delta_l^v(m)}{\rho_1} \right)^2 + \sum_{l \in \mathrm{IT}_i} \left(\hat{r}_l(m) - r_l(m) - \frac{\delta_l^v(m)}{\rho_1} \right)^2 \\
& + 1_{\{S(m), D(m)\}}(v) \left(\hat{r}_m^v - r_m - \frac{\delta_m^v}{\rho_1} \right)^2 \\
\text{s.t.} \quad & \sum_{l=(s,v) \in \mathcal{L}} \left(1_{\{\bigcup_{k \neq i} \mathcal{Z}_k\}}(s) \hat{r}_l^v(m) + 1_{\{\mathcal{Z}_i\}}(s) \hat{r}_l(m) \right) + 1_{\{S(m)\}}(v) \hat{r}^v(m) \\
& = \sum_{l=(v,d) \in \mathcal{L}} \left(1_{\{\bigcup_{k \neq i} \mathcal{Z}_k\}}(d) \hat{r}_l^v(m) + 1_{\{\mathcal{Z}_i\}}(d) \hat{r}_l(m) \right) + 1_{\{D(m)\}}(v) \hat{r}^v(m), \\
& \qquad \forall\, m = 1 \sim M.
\end{aligned}
$$

- No closed-form solution – efficient network optimization algorithms, e.g., relax code [Bertsekas 87]
- Pros: faster convergence rate, less info. exchange

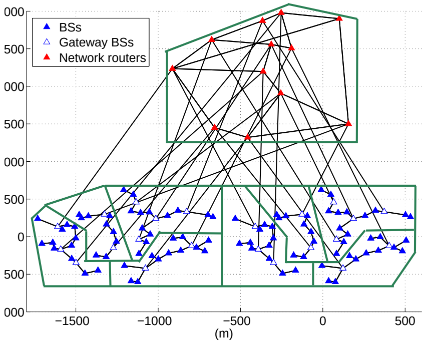
Numerical Experiment

- For each commodity, the source (destination) node is a randomly selected network router (BS)
- All simulation results are averaged over 100 realizations
- The ADMM stoping criterion is

  Maximum constraint violation $\leq 10^{-4}$

  Maximum relative increase of objective $\leq 10^{-3}$
- We set the stepsize $\rho_1 = 0.01$ if not stated
- Algorithm 1 is implemented in C language

Parallel Implementation



- The performance of Algorithm 1 is tested using the network topology provided by Huawei (114 BSs and 12 network routers)
- The BS nodes are split into 8 cores, and all RAN nodes belong to 1 core.

Parallel Implementation

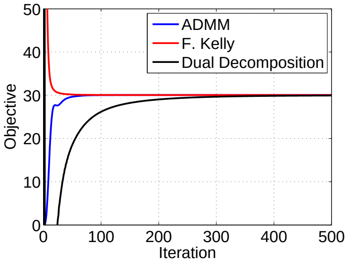
- The computation time vs the # of commodities (AMD Opteron 8356 2.3GHz)

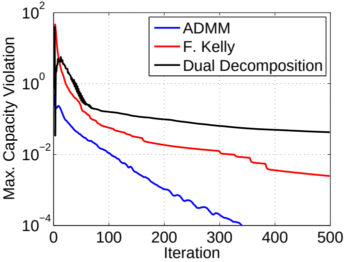
# of Commodities	50	100	200	300
Time (s) (Sequential)	1.04	2.03	4.73	8.53
Time (s) (Parallel)	0.20	0.37	0.75	1.10
Time (s) (Gurobi)	0.20	0.64	1.65	2.51
# of Variables	1.4×10^4	2.9×10^4	5.8×10^4	8.7×10^4
# of Constraints	2.1×10^4	4.2×10^4	8.4×10^4	1.3×10^5

- As the # of commodities increases, the efficiency improvement of parallel implementation increases.
- 2 times faster than commercial LP solver – Gurobi
- Time is approximately linear over # of commodities.

Comparison of Network Decomposition Algorithms

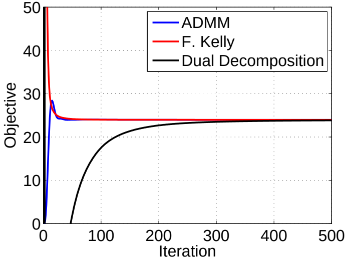
- Compare with i) dual decomposition [Chiang07] and ii) F. Kelly's algorithm [Kelly14]
- Random connected graph with 100 network routers.
  - Each network router connects to 3 network routers.
- Proportional fairness with path-flow formulation
  - Path for each commodity is the shortest path
- # of Commodities=50: $(\rho_1 = 0.5)$

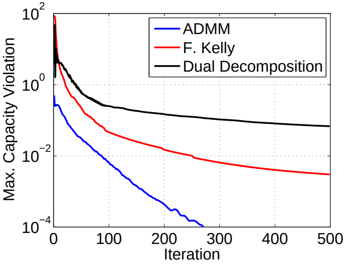




Comparison of Network Decomposition Algorithms

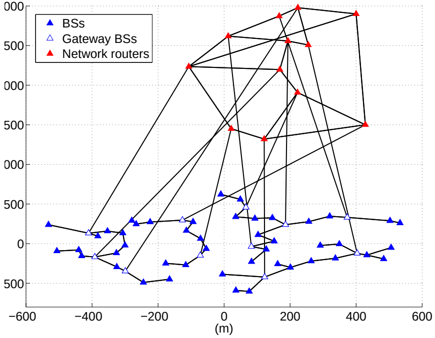
- # of Commodities=100: $(\rho_1 = 0.5)$





- Maximum capacity violation metric:
  - Algorithm 1 has faster convergence rate

Further Enhancement



- Acceleration enhancement of Algorithm 1 is tested using the tree network topology provided by Huawei (57 BSs and 12 network routers)

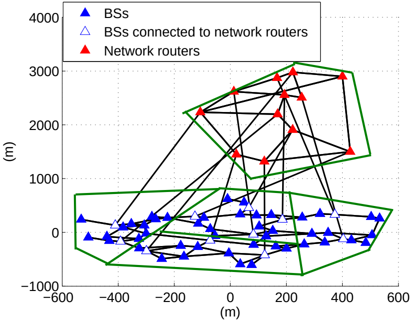
Further Enhancement (cont.)

- The # of iterations for Algorithm 1 with/without acceleration enhancement

# of Pairs	30	50	80	100	300
# of Iterations ($\rho_1 = 0.01$)	615	620	669	654	644
# of Iterations w/ dynamically adjust ρ ($\rho_1 = 0.001$)	293	293	318	306	300
# of Iterations w/ dynamically adjust ρ & over-relaxation ($\rho_1 = 0.001$)	285	285	298	291	281

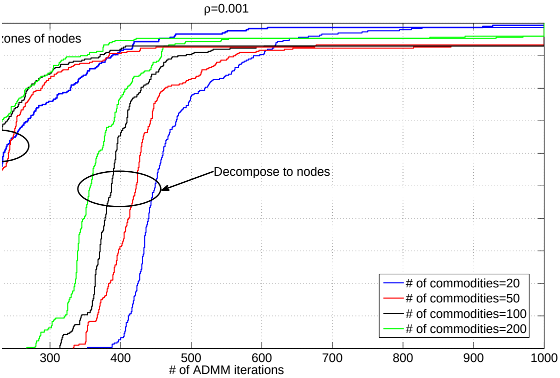
- More than 50% reduction in the # of iterations
- With relaxation method, up to additional 5% reduction.
- CPU times are similarly halved.

Algorithm 1 with Zoning



- Accelerate Algorithm 1 with zones of nodes is tested using the mesh network topology provided by Huawei (5 zones with 57 BSs and 12 network routers)

Algorithm 1 with Zoning (cont.)



- # of ADMM iterations significantly decreases – less slack variables

Agenda

- The Multi-Commodity Routing Problem
  - Brief Review of ADMM Algorithm
  - A Distributed ADMM Approach
- The Joint Power Allocation and Routing Problem
  - Algorithm Outline
  - An ADMM Approach for Updating $\{\mathbf{r}, \mathbf{p}\}$
- Numerical Results

Problem Formulation: Wireless Interfering Links

- Consider a more general problem that takes wireless interfering links into consideration
  - $\mathcal{U}$: the set of mobile users
  - K: # of orthogonal frequency tones in each wireless link
  - $\mathcal{L}^{wl} = \{(s, d, k) | s \in \mathcal{B}, d \in \mathcal{U}, k = 1 \sim K\}$: the set of wireless links
  - p_{ds}^{k}: scalar transmit precoder on link (s, d, k)
  - $r_l(m)$: rate on link l for commodity m
- Assume a per-BS power budget constraint

$$\sum_{k=1}^{K} \sum_{d:(s,d,k)\in\mathcal{L}^{wl}} |p_{ds}^{k}|^2 \leq \bar{p}_s, \ \forall \ s \in \mathcal{B} \tag{13}$$

Problem Formulation: Wireless Interfering Links (cont.)

- The rate constraint for a link $l = (s, d, k) \in \mathcal{L}^{wl}$ is

$$\sum_{m=1}^{M} r_l(m) \le \bar{r}_l(\mathbf{p}) \triangleq \log\left(1 + \frac{|h_{ds}^k|^2 |p_{ds}^k|^2}{\sum\limits_{\substack{(s',d',k)\in I(l)\\(s',d',k)\neq l}} |h_{ds'}^k|^2 |p_{d's'}^k|^2 + \sigma_d^2}\right) \quad (14)$$

  where

  - $\mathbf{p} \triangleq \{p_{ds}^k \mid \forall\, (s, d, k) \in \mathcal{L}^{wl}\}$
  - σ_d^2: the AWGN at receiver d,
  - $h_{ds}^k \in \mathbb{C}$: the wireless channel for $l = (s, d, k)$
  - $I(l) \subseteq \mathcal{L}^{wl}$: the set of links that interfere link l, i.e.,

$$I(l) \triangleq \{(s', d', k) \mid h_{ds'}^k \neq 0, (s, d, k) = l\}.$$

- The rate constraint is nonconvex with respect to $\mathbf{p}$!

Problem Setting for Wireless Links (cont.)

- **Task**: Jointly perform *1)*: routing of M commodity flows, and *2)* allocating transmit power on each wireless link.

$$\begin{aligned}(\mathcal{Q}):\ \max\quad & r\\ \text{s.t.}\quad & r \geq 0,\ r_m \geq r,\ r_l(m) \geq 0,\ m = 1 \sim M,\ \forall\, l \in \mathcal{L}\\ & (6),\ (7),\ (13),\ \text{and}\ (14).\end{aligned}$$

- Difficult joint optimization problem
  - Wireless link rate constraints (14) are nonconvex
  - Flow conservation constraints couple all variables
  - Multiple frequency tones and multiple antenna at BS makes the problem NP-hard [Razaviyayn 13]

The Proposed N-MaxMin WMMSE Algorithm

- To handle the nonconvex problem $(\mathcal{Q})$, we exploit the following rate-MSE relationship
- **Lemma** [Razaviyayn 13] For a given link $l = (s, d, k) \in \mathcal{L}^{wl}$, its rate $\bar{r}_l(\mathbf{p})$ can be equivalently expressed as

$$\bar{r}_l(\mathbf{p}) = \max_{u_l, w_l} 1 + \log(w_l) - w_l E_l(\mathbf{p}, u_l) \tag{15}$$

where

- For link $l = (s, d, k)$, the MSE at user :

$$E_l(\mathbf{p}, u_l) \triangleq 1 + \sigma_d^2 |u_l|^2 - 2Re\{u_l^* h_{ds}^k\} p_{ds}^k + \sum_{(s', d', k) \in I(l)} |u_l|^2 |h_{ds'}^k|^2 |p_{d' s'}^k|^2$$

- u_l: the receive beamformer
- w_l: the weighting coefficient of MSE

The Proposed N-MaxMin WMMSE Algorithm (cont.)

- Given the rate-MSE relationship, $\bar{r}_l(\mathbf{p})$ is replaced with fixed $\mathbf{u} \triangleq \{u_l \mid l \in \mathcal{L}^{wl}\}$ and $\mathbf{w} \triangleq \{w_l \mid l \in \mathcal{L}^{wl}\}$:

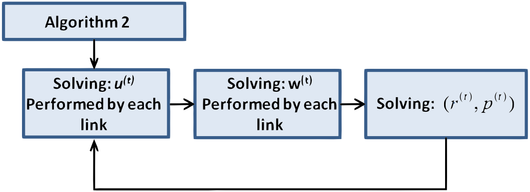
$$
\begin{aligned}
(\hat{\mathcal{Q}}): \ \max \quad & r \\
\text{s.t.} \quad & r \geq 0,\ r_m \geq r,\ r_l(m) \geq 0,\ m = 1 \sim M,\ \forall\, l \in \mathcal{L} \\
& \sum_{m=1}^{M} r_l(m) \leq 1 + \log(w_l) - w_l E_l(\mathbf{p}, u_l), \forall\, l \in \mathcal{L}^{wl},\ \text{(quadratic)} \\
& \text{(6), (13), and (7)}
\end{aligned}
$$

$\Rightarrow$ Convex for each $\mathbf{u}$, $\mathbf{w}$, or $\{\mathbf{r}, \mathbf{p}\}$ while fixing others

$\Rightarrow$ Propose to iteratively update $\mathbf{u}$, $\mathbf{w}$, and $\{\mathbf{r}, \mathbf{p}\}$

The Proposed N-MaxMin WMMSE Algorithm (cont.)

- Outline of the proposed N-MaxMin WMMSE Algorithm (Algorithm 2)



- **Theorem** The sequence $\{\mathbf{r}^{(t)}, \mathbf{p}^{(t)}\}$ generated by Algorithm 2:
  - converges to the stationary solution of problem $(\mathcal{Q})$
  - every global optimal solution of problem $(\mathcal{Q})$ corresponds to a global optimal solution of the reformulated problem $(\hat{\mathcal{Q}})$ with the same objective value

Updating Steps for Each Variable Set

- Fixing $\mathbf{p}$, the optimal $\mathbf{u}$ (resp. $\mathbf{w}$) is obtained in closed-form for any tuple $l = (s, d, k) \in \mathcal{L}^{wl}$:

$$u_l = \left(\sum_{(s',d',k) \in I(s,d,k)} | h^k_{ds'}|^2 |p^k_{d's'}|^2 + \sigma_d^2 \right)^{-1} h^k_{ds} p^k_{ds},$$

$$w_l = \left(1 - (h^k_{ds} p^k_{ds})^* u_l\right)^{-1}.$$

$\Rightarrow$ Decouple over each wireless link

- Fixing $\mathbf{u}$ and $\mathbf{w}$ problem $(\hat{\mathcal{Q}})$ is a large convex problem with coupled flow conservation constraints

$\Rightarrow$ Apply ADMM algorithm again!

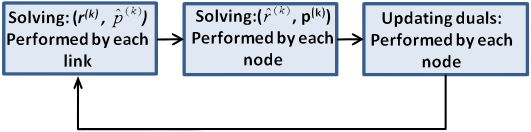
An ADMM Approach for Updating $\{\mathbf{r}, \mathbf{p}\}$

- To decouple the constraints, the following slack variables besides (12) are introduced

$$r_l(m) = r_l^s(m),\ r_l(m) = r_l^d(m),\ m = 1 \sim M,\ \forall\ l = (s, d, k) \in \mathcal{L}^{wl},$$
$$p_{d's',ds}^k = p_{ds}^k,\ \forall\ (s, d, k), (s', d', k) \in \mathcal{L}^{wl},\ \forall\ (s, d, k) \in I(s', d', k).$$

  - With such variable splitting, each variable $p_{d's',ds}^k$ will only appear in a single rate-MSE constraint

- The outline of the proposed ADMM approach for $\{\mathbf{r}, \mathbf{p}\}$ is given below



where $\hat{\mathbf{p}} \triangleq \{p_{d's',ds}^k \mid \forall (s, d, k) \in \mathcal{L}^{wl},\ (s, d, k) \in I(s', d', k)\}$.

An ADMM Approach for Updating $\{\mathbf{r}, \mathbf{p}\}$ (cont.)

- Similar to Algorithm 1, the proposed ADMM approach for updating $\{\mathbf{r}, \mathbf{p}\}$ has the following properties
  - Efficiency — (semi)closed-form and solvable in parallel
  - Distributed implementation with local information exchange
- Moreover, problem $(\mathcal{Q})$ can be extended to include per-commodity QoS requirements
  - Commodity q of a set $\mathcal{Q}$ is required to satisfy minimum rate $\underline{r}_q$
  - $r_m \geq 0, \ \forall\, m$, is modified as $\begin{cases} r_m \geq \underline{r}_q, & \forall\, m \in \mathcal{Q} \\ r_m \geq r, & \forall\, m \in \mathcal{Q}^c \end{cases}$.
  - The proposed Algorithm 2 can then be easily modified.

Numerical Experiment

- The network topology is the same as the tree topology in the routing problem (57BSs and 12 network routers)
- Each frequency tone has 1MHz, and number of $K = 3$
- The channel is generated as $CN(0, (200/d)^3)$ where d is the distance between BS and user
- ρ_1 is chosen as 0.1
- The termination criterion is chosen the same as the routing problem

Numerical Experiment (cont.)

- For comparison, the following two heuristics are used
- **Heuristic 1**:
  (a) Each user chooses the BS having strongest channel and frequency pair with itself as the serving BS
  (b) Each BS uniformly allocates its power budget to each served user while the power budget for each frequency is uniformly allocated.
  (c) With this fixed power allocation, the capacity of the wireless links are available
  (d) Maximize the minimum achievable rate with predetermined optimal routing

Numerical Experiment (cont.)

- **Heuristic 2**:
  (a) Each BS uniformly allocates its power budget to each orthogonal subcarrier tone
  (b) The max-min problem is solved under additional interference-free constraint

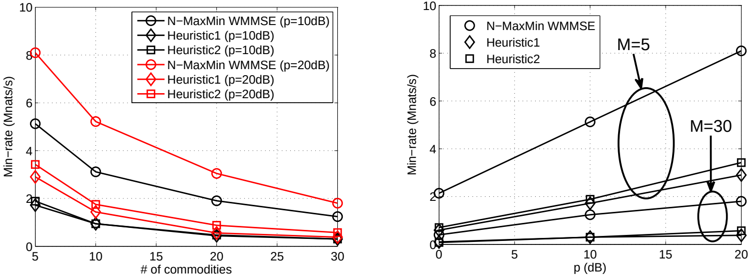
$$
\begin{aligned}
&\max\ r \\
&\text{ s.t. } r_m \geq r,\ \ r_l(m) \geq 0,\ \ m = 1 \sim M,\ \ \forall l \in \mathcal{L} \\
&\qquad \sum_{m=1}^{M} r_l(m) \leq \alpha_l \log\left(1 + \frac{|h_{ds}^k|^2 \bar{p}_s / K}{\sigma_d^2}\right),\ \forall\, l = (s,d,k) \in \mathcal{L}^{wl} \\
&\qquad \sum_{n \in I(l)} \alpha_n = 1,\ \ \alpha_l \in \{0,1\},\ \forall\, l,\ n \in \mathcal{L}^{wl},
\end{aligned}
$$

(6) and (7).

(c) The integer constraint is hard $\Rightarrow$ relax to $\alpha_l = [0,1]$

Numerical Experiment (cont.)

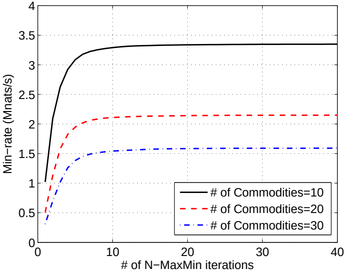
- Each mobile user is served only by BSs within 300 meters while being interfered by all BSs.
- More than twice of minimum rate of the heuristic algorithms.

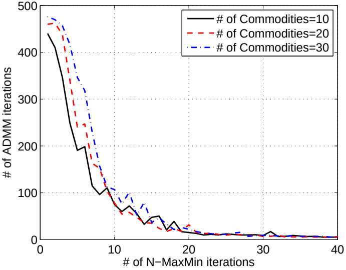


$\Rightarrow$ Proper power allocation is needed for problem $(\mathcal{Q})$.

Numerical Experiment (cont.)

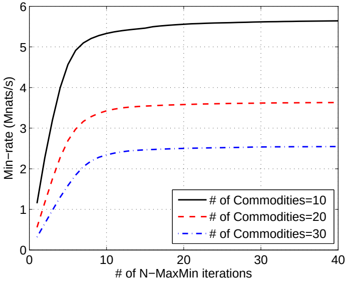
- Each mobile user is served only by BSs within 300 meters while being interfered by BSs within 800 meters.
- Power budget for each BS: 10dB, $\rho_2 = 0.005$ (for precoder variables)

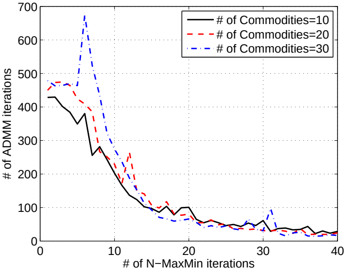




Numerical Experiment (cont.)

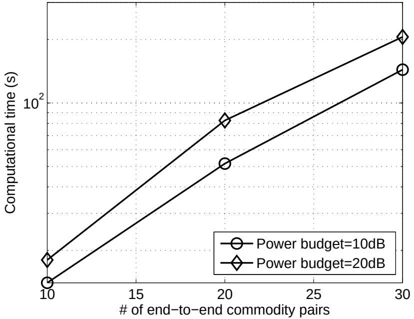
- Each mobile user is served only by BSs within 300 meters while being interfered by BSs within 800 meters.
- Power budget for each BS: 20dB, $\rho_2 = 0.001$ (for precoder variables)





Numerical Experiment (cont.)

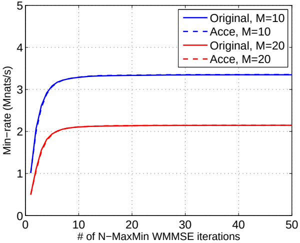
- Computational time for the first 10 N-MaxMinWMMSE iterations:

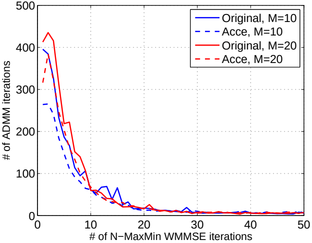


$\Rightarrow$ Within 3.5 minutes for all considered scenarios without exploiting parallel programming

Further Enhancement

- Apply the acceleration enhancement of ADMM
- Power budget for each BS: 10dB





- # of inner iterations decrease – especially in the first few iterations

Final Remarks

- Joint provision of backhaul and RAN offers great potential to improve user QoS and network throughput.
- For routing only problem, we develop a distributed/parallel algorithm; two times faster than commercial solvers
- For joint routing and power allocation problem, we exploit the rate-MSE relationship, and develop an algorithm capable of computing a high-quality solution in a distributed/parallel manner
- Joint provision can more than double the network performance.

Future Directions

A gold mine of challenging optimization problems – much more remains to be done

- Reduce the message passing between RAN and Backhaul
- Asynchronous updates: Backhaul TE updates at much slower rate than the RAN
- Joint processing among BSs, network caching, network coding
- Reduce the CSI requirement: expected sum-utility maximization
- Stochastic formulation of network provisioning to account for traffic dynamic

Reference

1 [Andrews 13] J. Andrews, "Seven ways that hetnets are a cellular paradigm shift," IEEE Communications Mag., vol. 51, no. 3, pp. 136-144, Mar. 2013.

2 [Bertsekas 87] D. P. Bertsekas, P. Hosein, and P. Tseng, "Relaxation methods for network flow problems with convex arc costs," SIAM Journal on Control and Optimization, vol. 25, no. 5, pp. 1219-1243, Sep. 1987.

3 [Bertsekas 88] D. P. Bertsekas and P. Tseng, "The relax codes for linear minimum cost network flow problem," Annals of Operations Research, vol. 13, no. 1, pp. 125-190, 1988.

4 [Bjornson 13] E. Bjornson and E. Jorswieck, "Optimal resource allocation in coordinated multi-cell systems," Foundations and Trends in Communications and Information Theory, vol. 9, no. 2-3, pp. 113-381, 2013.

5 [Chiang 07] M. Chiang, S. Low, A. R. Calderbank, and J. C. Doyle, "Layering as optimization decompositioon: A mathematical theory of network architectures," Proceedings of the IEEE, vol. 95, no. 1, pp. 255-312, Jan. 2007.

Reference

6 [Chowdhery 11] A. Chowdhery, W. Yu, and J. M. Cioffi, "Cooperative wireless multicell ofdma network with backhaul capacity constraints," in Proc. of IEEE International Conference on Communications, Jun. 2011, pp. 1-6.

7 [Kelly 14] F. Kelly, "Global optimization of network flows," preprint, 2014.

8 [Gesbert 10] D. Gesbert, S. Hanly, H. Huang, S. Shamai (Shitz), O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," IEEE Journal on Selected Areas in Communications, vol. 28, no. 9, pp. 1380-1408, Dec. 2010.

9 [Hong 12] M. Hong and Z.-Q. Luo, "Signal processing and optimal resource allocation for the inter- ference channel," EURASIP E-Reference Signal Processing, 2012, accepted, available at http://arxiv.org.

10 [Razaviyayn 13] M. Razaviyayn, M. Hong, and Z.-Q. Luo, "Linear transceiver design for a MIMO interfering broadcast channel achieving max-min fairness," Signal Processing, vol. 93, no. 12, pp. 3327-3340, Dec. 2013.

Reference

11 [Shroff 06] N. Shroff and R. Srikant, "A tutorial on cross-layer optimization in wireless networks," IEEE Journal on Selected Areas in Communications, vol. 24, no. 8, pp. 1452-1463, Aug. 2006.

12 [Simeone 09] O. Simeone, O. Somekh, H. Poor, and S. Shamai (Shitz), "Downlink multicell processing with limited-backhaul capacity," EURASIP Journal on Advanced Signal Processing, vol. 2009, pp. 1-10, Feb. 2009.

13 [Xiao 04] L. Xiao, M. Johansson, and S. P. Boyd, "Simultaneous routing and resource allocation via dual decomposition," IEEE Transactions on Communications, vol. 52, no. 7, pp. 1136-1144, Jul. 2004.

14 [Zakhour 11] R. Zakhour and D. Gesbert, Optimized data sharing in multicell MIMO with finite backhaul capacity, IEEE Transactions on Signal Processing, vol. 59, no. 12, pp. 6102-6111, Dec. 2011.