Control over Wireless Networks

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Wireless networked control system
Wireless networked control systems

Motivating Control Applications

Truck platooning

Industrial process control
Air Drag Reduction in Truck Platooning


Wireless Control of Truck Platoons

Rapport on vehicle platooning developed by KTH and Scania (Oct, 2011)

PhD student Assad Alam on Discovery Channel (Jan, 2012)
Off- and On-Board Wireless Control

**Off-board transport planner**
- Monitor trucks and traffic
- Choose routes to maximize platooning
- Replan due to new trucks, weather, changing traffic conditions, etc

**On-board platoon coordinator**
- Coordinate platoon creation, merge, split etc
- Optimize platoon speed
- Interact with cruise controllers
Communication in process industry

Wireless systems benefit from
- Lower installation and maintenance costs
- Increased sensing capabilities and flexibility

Major consequence for control system architecture

Today’s industrial communication architecture

Centralized control system with low-level loops closed over wired network
Towards a wireless network control systems architecture

- Local control loops closed over **wireless multi-hop network**
- Potential for a dramatic change:
  - From fixed hierarchical centralized system to flexible distributed
  - High-level optimization vs low-level real-time control
  - 4G-5G?

Radio Channel Measurements in Industrial Environment

- Rolling mill at Sandvik in Sweden
- Study of 2.45 GHz radio channel properties
- Slow but substantial RSSI variations due to mobile machines

Ahlen et al, 2012
What is the effect on control performance of a shared wireless network?

- A real experiment with water tanks and no MA:
- Let’s compare results of reference tracking

Packet loss influence on control performance
Where to take medium access decisions?

Sensor node makes local decisions on when to communicate

Network manager allocates communication slots

Controller requests sensor data

Is there a separation principle for scheduling-estimation-control?
Stochastic control formulation

Plant:
\[ x_{k+1} = Ax_k + Bu_k + w_k \]

Scheduler:
\[ \delta_k = f_k(I_k^S) \in \{0,1\} \]
\[ I_k^S = \left( \begin{array}{c} x_k^0, x_{k-1}^0, \delta_0^{k-1}, \delta_0^{k-1} \end{array} \right) \]

Controller:
\[ u_k = g_k(I_k^C) \]
\[ I_k^C = \left( \begin{array}{c} y_k^0, \delta_0^k, \delta_0^{k-1} \end{array} \right) \]

Cost criterion:
\[ J(f, g) = E[x_0^T Q_0 x_N + \sum_{k=0}^{N-1} (x_k^T Q_1 x_k + u_k^T Q_2 u_k)] \]

Conditions for Certainty Equivalence

**Corollary:** The optimal controller for the system \( \{\mathcal{P}, S(f), C(g)\} \), with respect to the cost \( J \), is certainty equivalent if and only if the scheduling decisions \( \gamma \) are not a function of the applied controls \( u \).
Architecture with State-based Scheduler

- **Plant** $\mathcal{P}$:
  \[ x_{k+1} = ax_k + bu_k + w_k \]
- **State-based Scheduler** $\mathcal{S}$:
  \[ \gamma_k = \begin{cases} 
  1, & |x_k - x_{\gamma_k}|^2 > \epsilon_d, \\
  0, & \text{otherwise.}
  \end{cases} \]
  \[ x_{\gamma_k-1} = ax_{\gamma_k-1} + bu_{\gamma_k-1} \]
- **CRM**:
  \[ P(\alpha_k|\gamma_k=1) = P(\alpha_k'|\gamma_k=1) = p_\alpha, \]
  \[ \delta_k = \alpha_k(1 - \alpha_k^N) \]
- **Observer** $\mathcal{O}$:
  \[ y_k^0 = \delta_k^x x_k^0 \]
  \[ x_k^0 = \delta_k^x (ax_{\gamma_k-1} + bu_{\gamma_k-1}) + \delta_k x_k \]
- **Controller** $\mathcal{C}$:
  \[ u_k = -L^2 x_k \]

Communicate when there is new plant information

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Event-based control of an integrator

**Plant**
\[ dx_t = dW_t + u_t dt, \quad x(0) = x_0, \]

**Sampling events**
\[ \mathcal{T} = \{ \tau_0, \tau_1, \tau_2, \ldots \} \]

**Impulse control**
\[ u_t = \sum_{n=0}^{\infty} x_n \delta(\tau_n) \]

**Average sampling rate**
\[ R_T = \limsup_{M \to \infty} \frac{1}{M} E \left[ \int_0^M \sum_{n=0}^{\infty} 1_{\{\tau_n \leq M\}} \delta(s - \tau_n) \, ds \right] \]

**Average cost**
\[ J = \limsup_{M \to \infty} \frac{1}{M} E \left[ \int_0^M x_n^2 \, ds \right] \]
Event-based control

Equidistant Lebesgue sampling with level set

\[ \mathcal{L}^* = \{ k \Delta | k \in \mathbb{Z} \} \]

and communication events

\[ \tau = \inf \{ \tau | \tau > \tau_i, \ x_\tau \in \mathcal{L}, \ x_\tau \notin \mathcal{L} \} \]

Average sampling rate

\[ R_\Delta = \frac{1}{\mathbb{E}[\tau_\Delta]} = \frac{1}{\Delta^2} \]

Average cost

\[ J_\Delta = \frac{\mathbb{E} \left[ \int_0^T x_\tau^2 ds \right]}{\mathbb{E}[\tau_\Delta]} = \frac{\Delta^2}{6} \]

Influence of communication losses

Times when packets are successfully received

\[ \rho_i \in \{ \tau_0 \leq \tau_1 \leq \tau_2 \leq \ldots \} \]

\[ \rho_i \geq \tau_i \]

Average rate of packet reception with IID prob \( p \) packet loss:

\[ R_p = \lim_{M \to \infty} \frac{1}{M} \mathbb{E} \left[ \sum_{s=0}^{M-1} 1_{(s+1)\Delta \leq \rho_s} \right] = p \cdot R_\Delta \]

Define the times between successful packet receptions

\[ \rho_{(p, \Delta)} \]

Average cost

\[ J_p = \lim_{T \to \infty} \frac{1}{T} \mathbb{E} \left[ \int_0^T x_\tau^2 ds \right] = \frac{\mathbb{E} \left[ \int_0^T x_\tau^2 ds \right]}{\mathbb{E}[\rho_{(p, \Delta)}]} \]
**Event-based control under packet losses**

**Proposition**
If packet losses are IID with prob $p$, then equidistant event-based $\Delta$-sampling control gives average cost

$$J_p = \frac{\Delta^2 (5p + 1)}{6 (1 - p)}$$

**Remark**
- Event-based control with losses always better than periodic with losses.
- Event-based control with losses outperformed by periodic control without losses if

$$\frac{(1 + 5p)}{3(1 - p)} \geq 1$$

so if $p \geq 0.25$ then periodic sampling do better than event-based sampling.

Rabi and J., 2009

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**Sensor data ACK’ s**

If controller perfectly acknowledges packets to sensor, then event detector can adjust its sampling strategy

Let

$$\Delta (i) = \sqrt{i + 1} \Delta_0$$

where $i \geq 0$ number of samples lost since last successfully transmitted packet

Gives $E \left[ r_{i+1} - r_i \right]$ independent of $i$.

Better performance than fixed $\Delta (i)$ for same sampling rate:

$$J_p^i = \frac{\Delta^2 (1 + p)}{6 (1 - p)} \leq \frac{\Delta^2 (1 + 5p)}{6 (1 - p)} = J_p.$$

Rabi and J., 2009
Conclusions

- **Traditional control** design is based on **perfect information** being **periodically** circulated in the system.

- Wireless control applications have **diverse requirements**
  - Time constants from milliseconds to hours
  - From safety-critical to best effort systems
  - Worst-case vs average performance

- **Theory for wireless control** design taking into account
  - Models and optimization of the wireless network
  - Adaptive resource management based on control attention
  - Hybrid aperiodic (event-based) and periodic control

http://people.kth.se/~kallej
Hybrid control for hybrid MAC

Utilize that hybrid MAC has both contention-free period (CFP) and contention access period (CAP):

**Example** Disturbance rejection in one of three plants

Control over CAP

Control over CFP

Control over CAP-CFP

Araujo et al., 2009